

# New resummation techniques of divergent series: the Painlevé equation PII

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## Abstract

Most differential equations have asymptotic series which are divergent; for example, in physics, most Hamiltonian perturbation expansions are divergent, only being accurate to a small number of terms. Resummation methods are used to sum divergent series and obtain actual solutions. We are interested in resummation methods that give maximum information about the associated function even when truncating the divergent asymptotic series.

One of the most prevalent resummation methods is Borel summation; however, Borel summation is very limited in its applicability to equations. In particular, it always fails along half-lines in which the terms of the divergent series have the same phase, known as Stokes lines. In the last few years, mathematicians have developed new, efficient methods of resummation which employ the property of resurgence, a remarkable property of divergent series of natural origin which has been recently discovered. One can use this property to deal with incomplete information such as partial perturbation expansions.

Our group has developed such a method, applying it to the Painlevé equation PII (from now on, PII) to test its efficiency and applicability for resumming divergent series. Accuracy increased greatly from the usual Borel-Padé approximations of a series used in physics by using asymptotics of continued fractions as well as using Padé with conformal mappings of the complex plane. Using this, we rediscovered the connection constant (a property of the underlying function) and approximated the behavior of the function at small values of  $t$  (large-to-small coupling).

This new method is more efficient in resumming divergent series and dealing with incomplete information. Moving forward, we will calculate the binary rational expansion, a better convergent expansion of the solution, for PII. This study is directly connected to random matrix theory in physics. We also hope to apply the method to other problems in mathematics and physics, such as obtaining higher precision in critical expansions at low and high temperatures for problems such as the 3-dimensional Ising model in statistical mechanics.