# Math 128A: Worksheet #2

Name: _____ Date: September 21, 2020

Fall 2020

**Problem 1** (2.4 #9)**:** a. Construct a sequence that converges to 0 of order 3.

b. Suppose $\alpha > 1$. Construct a sequence that converges to 0 of order $\alpha$.

**Problem 2** (2.5 #2)**:** Consider the function $f(x) = e^{6x} + 3(\ln 2)^2 e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3$. Use Newton's method with $p_0 = 0$ to approximate a zero of $f$. Generate terms until $|p_{n+1} - p_n| < 0.0002$. Construct the sequence $\{\hat{p}_n\}$. Is the convergence improved?

*Extra:* Using $g(x) = x - \frac{f(x)}{f'(x)}$, use Steffenson's method to find the zero of $f$. Is convergence improved?

**Problem 3** (2.5 #15)**:** Suppose that $\{p_n\}$ is superlinearly convergent to $p$. Show that

$$\lim_{n\to\infty} \frac{|p_{n+1} - p_n|}{|p_n - p|} = 1.$$

*Reminder:* A sequence $\{p_n\}$ is said to be superlinearly convergent to $p$ if

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|} = 0.$$

**Problem 4** (3.1 #1c)**:** For the function $f(x) = \sqrt{1+x}$, let $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(0.45)$ and find the absolute error.

**Problem 5** (3.1 #3)**:** Use Theorem 3.3 to find an error bound for the approximations in the previous exercise.