Math 128A: Worksheet #4

Name: ______________________________ Date: October 5, 2020

Fall 2020

Problem 1.

1. Use the Hermite theorem or divided differences to construct an approximating polynomial for the following data:

x	$f(x)$	$f'(x)$
0	1	0
0.5	0.8	-0.64

2. This data was taken from the function $f(x) = (1 + x^2)^{-1}$. Use the interpolating polynomials to approximate $f(0.25)$. Give the actual error and the error bound on this approximation.

Problem 2.

1. Construct the natural cubic spline for the following data (by hand and using Matlab):

x	$f(x)$
0	3
1	0
2	3

2. This data was taken from the function $f(x) = 3(x-1)^2$. Use the cubic splines to approximate $f(0.5)$ and $f'(0.5)$, and calculate the actual error.

3. This data also matches the function $g(x) = 3x^4 - 5x^3 - 3x^2 + 2x + 3$. Use the cubic splines to approximate $g(0.5)$ and $g'(0.5)$, and calculate the actual error.

Problem 3 (4.1, #28)**.** Derive a method for approximating $f'''(x_0)$ whose error term is of order h^2 by expanding the function f in a fourth Taylor polynomial about x_0 and evaluating at $x_0 \pm h$ and $x_0 \pm 2h$.

Problem 4 (4.2, #1a)**.** Apply Richardson's Extrapolation on the centered-difference formula:

$$f'(x_0) = \frac{1}{2h}[f(x_0+h) - f(x_0-h)] - \frac{h^2}{6}f'''(x_0) - \frac{h^4}{120}f^{(5)}(x_0) - ...$$

to determine $N_3(h)$, an approximation to $f'(x_0)$, for the following function and stepsize:

$$f(x) = \ln(x), \quad x_0 = 1.0, \quad h = 0.4.$$