

Math 128A: Worksheet #6

Name: _____ Date: October 12, 2020

Fall 2020

Problem 1. Consider the following numerical integration rule:

$$\int_a^b f(x) dx \approx (b-a) \left(\frac{1}{4} f(a) + \frac{3}{4} f\left(a + \frac{2}{3}(b-a)\right) \right)$$

What is the degree of accuracy of this integration rule?

Hint: In order to make the computations simpler, you can assume without loss of generality that $a = 0$ and $b = 1$.

Problem 2. Consider a function $f : [0, 1] \rightarrow \mathbb{R}$. We want to approximate the integral $I = \int_0^1 f(x) dx$ using composite numerical integration based on the above integration rule. Let $I(h)$ denote the approximation of I we obtain by dividing the interval $[0, 1]$ into subintervals of length h . What is the order of the error $|I - I(h)|$ as $h \rightarrow 0$, i.e. what is the largest integer k such that

$$|I - I(h)| = \mathcal{O}(h^k) \text{ as } h \rightarrow 0$$

Hint: In each of the small subintervals of length h approximate f by a Taylor polynomial and use the degree of accuracy determined in Problem 1.