Math 128A: Worksheet #7

Name: _____ Date: October 19, 2020

Fall 2020

Problem 1 (4.6 #1 c,d)**.** Compute Simpson's rule approximations $S(a,b), S\left(a, \frac{a+b}{2}\right)$, and $S\left(\frac{a+b}{2}, b\right)$ for the following integrals and verify the estimate given in the approximation formula:

c. $\displaystyle\int_0^{0.35} \frac{2}{x^2-4}\,dx$

d. $\displaystyle\int_0^{\pi/4} x^2 \sin x\,dx$

Problem 2. Let $I(a,b)$ and $I\left(a, \frac{a+b}{2}\right) + I\left(\frac{a+b}{2}, b\right)$ denote the single and double applications of the Simpson's Three-Eighths rule to $\int_a^b f(x)\,dx$. That is,

$$I(a,b) = \frac{3h}{8}[f(a) + 3f(a+h) + 3f(a+2h) + f(b)],$$

where $h = \frac{b-a}{3}$. $I\left(a, \frac{a+b}{2}\right)$ and $I\left(\frac{a+b}{2}, b\right)$ are defined similarly.

Derive the relationship between

$$\left| I(a,b) - I\left(a, \frac{a+b}{2}\right) - I\left(\frac{a+b}{2}, b\right) \right|$$

and

$$\left| \int_a^b f(x)\,dx - I\left(a, \frac{a+b}{2}\right) - I\left(\frac{a+b}{2}, b\right) \right|.$$

What does this tell us about estimating the error of our numerical integration?