Math 128A: Worksheet #8

Name: ____________________ Date: October 26, 2020

Fall 2020

Problem 1. Consider the integration rule

$$\int_0^1 f(x)\,dx \approx \sum_{i=1}^{n} c_i f(x_i)$$

with n nodes $x_1 < \cdots < x_n$ and n weights $c_1, \ldots, c_n$.

(a) First, suppose that the nodes $x_1, \cdots, x_n$ are fixed. Show that by choosing the weights $c_1, \ldots, c_n$ appropriately we can always guarantee the degree of precision is at least $n-1$.

(b) What is the highest degree of precision we can possibly achieve with n nodes and weights? Show that it is impossible to have degree of precision higher than that.

Problem 2. Approximate the integral

$$\int_{-1}^{1}\int_{-1}^{1}(x^2+y^2)\;dx\;dy$$

using the composite trapezoidal rule with $n = 2$ subintervals in both the x and y direction.

Problem 3. (a) The error term of approximating the integral $\int_a^b f(x)\,dx$ using composite Simpson's rule is given by

$$-\frac{b-a}{180}h^4 f^{(4)}(\mu)$$

where h denotes the length of the subintervals into which $[a,b]$ is divided. In order to compute an approximation of the integral via composite Simpson's rule we need to evaluate the function f a certain number of times. Call this number N. Express N in terms of h. How does the error depend on N?

(b) The error term for approximating the double integral $\int_a^b \int_c^d f(x,y)\,dx\,dy$ using double Simpson's rule is given by

$$-\frac{(d-c)(b-a)}{180}h^4 \left(\frac{\partial^4 f}{\partial x^4} f(\eta,\mu) + \frac{\partial^4 f}{\partial y^4} f(\eta',\mu') \right).$$

Here the length of the subintervals in both x and y direction is given by h. Again, let N denote the number of times we need to evaluate f in order too compute the approximation. Repeat the same exercise. Express N in terms of h and the error in terms of N.

(c) What do you observe? What problem might we encounter when integrating a function $f(x_1,\ldots,x_n)$ on a high dimensional domain?

Problem 4 (4.8, #9-ish)**.** Use Algorithm 4.4 (Simpson's Double Integral) with $n = m = 14$ to approximate

$$\int\int_R e^{-(x+y)}\, dA$$

for the region R in the plane bounded by the curves $y = x^2$ and $y = \sqrt{x}$.