Math 128A: Worksheet #9

Name: ____________________________________ Date: November 2, 2020

Fall 2020

Problem 1 (4.9, #1c)**.** Use the Composite Simpson's rule with $n = 8$ to approximate

$$\int_1^2 \frac{\ln x}{(x-1)^{1/5}}\, dx.$$

Problem 2. Consider a function $f : \mathbb{R} \to \mathbb{R}$ that is continuous and differentiable. Show that $|f'(x)| \leq L$ for all $x \in \mathbb{R}$ if and only if f is Lipschitz continuous with Lipschitz constant L.

Problem 3. Show that if $f : \mathbb{R} \to \mathbb{R}$ is Lipshitz continuous, then f is continuous.

Problem 4 (5.1, #4b)**.** Let $f(t, y) = \dfrac{1+y}{1+t}$.

1. Does f satisfy a Lipschitz condition on $D = \{(t, y) : 0 \leq t \leq 1, -\infty < y < \infty\}$.
2. Can Theorem 5.4 and 5.6 be used to show that the initial value problem

$$y' = f(t, y), \quad 0 \leq t \leq 1, \quad y(0) = 1,$$

has a unique solution and is well-posed?