Math 128A: Worksheet #10

Name: _____ Date: November 9, 2020

Fall 2020

Problem 1. Consider the initial value problem

$$\begin{cases} y'(t) = y(t) \\ y(0) = y_0 \end{cases}$$

1. Determine the exact solution of this initial value problem
2. Apply one step with stepsize $h > 0$ of each of the following methods (look them up in Chapter 5.4 of the textbook): Euler's method, Midpoint method, Modified Euler's method (Explicit Trapezoidal rule), Heun's method, and the Runge-Kutta Order Four method.
3. Compute the local truncation error of these methods. What is the order of the local truncation error as $h \to 0$?

Problem 2. Now consider the differential equation $y'(t) = f(t, y(t))$ where f is smooth (infinitely differentiable).

1. Show that the local truncation error of Euler's method is order $\mathcal{O}(h)$.
2. Show that the local truncation error of Modified Euler's method (Explicit Trapezoidal rule) is order $\mathcal{O}(h^2)$.

Hint: compute Taylor expansions with respect to h.

Problem 3. Consider the second order initial value problem

$$\begin{cases} y''(t) + \sin(y'(t)) + y(t)^2 = t^2 \\ y(0) = 1 \\ y'(0) = \pi/2 \end{cases}$$

1. Convert this second order equation into a first order system of equations.
2. Apply one step of Euler's method with step size h to this first order system.