Math 128A: Worksheet #12

Name: _____   Date: <u>November 23, 2020</u>

<u>Fall 2020</u>

Problem 1. The Implicit Midpoint method for solving a differential equation $y'(t) = f(t, y(t))$ is given by

$$w_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, \frac{w_i + w_{i+1}}{2}\right).$$

Show that the Implicit Midpoint method is A-stable.

Problem 2. Consider the following system of linear equations

$$\begin{cases} x_1 + x_2 - x_3 = 0 \\ \quad\quad 12x_2 - x_3 = 4 \\ 2x_1 + x_2 + x_3 = 5 \end{cases}$$

Solve this system using Gauss elimination and Gauss elimination with partial pivoting. How many row interchanges do you need in each case?

Problem 3. Let A and B be $\ell \times m$ matrices and C be a $m \times n$ matrix. How many additions and multiplications are necessary to compute $A + B$ and AC if we compute the sum and the product directly following the definition?

Problem 4. Let A and B be two $m \times m$ matrices and suppose that AB is invertible. Show that both A and B are invertible.