Math 128A: Worksheet #2

Name: _____ Date: February 3, 2021

Spring 2021

Problem 1 (Section 1.3, #7a)**.** Find the rate of convergence of the following function as $h \to 0$:

$$\lim_{h \to 0} \frac{\sin(h)}{h} = 1.$$

Problem 2 (Section 2.1, #17)**.** Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-4} to the solution of $x^3 - x - 1 = 0$ lying in the interval $[1, 2]$ using the bisection method.

Problem 3 (Section 2.2, #9)**.** Use Theorem 2.3 to show that $g(x) = \pi + 0.5\sin(x/2)$ has a unique fixed point on $[0, 2\pi]$. Use fixed-point iteration to find an approximation to the fixed point that is accurate to within 10^{-2}. Use Corollary 2.5 to estimate the number of iterations required to achieve 10^{-2} accuracy and compare this theoretical estimate to the number actually needed.

Problem 4. Consider the following two functions:

$$g_1(x) = -\frac{1}{12}x^3 + x + \frac{5}{12}$$
$$g_2(x) = \frac{2}{3}x + \frac{5}{3}\frac{1}{x^2}$$

Both have $x^* = \sqrt[3]{5}$ as a fixed point. For which of these functions does fixed point iteration converge to x^*? If both of them converge, which one is faster?