Math 128A: Worksheet #3

Name: _____ Date: February 10, 2021

Spring 2021

Problem 1 (2.3 #1)**:** Let $f(x) = x^2 - 6$ and $p_0 = 1$. Use Newton's method to find p_2.

Problem 2 (2.3 #5a)**:** Use Newton's method to find a solution accurate to within 10^{-4} for:

$$x^3 - 2x^2 - 5 = 0, \quad [1, 4]$$

Problem 3 (2.4 #9)**:** a. Construct a sequence that converges to 0 of order 3.

b. Suppose $\alpha > 1$. Construct a sequence that converges to 0 of order α.

Problem 4: Consider the function $f(x) = x^4 + x^2$. Use Newton's method with $p_0 = 1$ to approximate a zero of f. Generate terms until $|p_{n+1} - p_n| < 0.0002$. Construct the sequence $\{\hat{p}_n\}$. Is the convergence improved?

Extra: Using $g(x) = x - \frac{f(x)}{f'(x)}$, use Steffenson's method to find the zero of f. Is convergence improved?

Problem 5 (2.5 #15)**:** Suppose that $\{p_n\}$ is superlinearly convergent to p. Show that

$$\lim_{n\to\infty} \frac{|p_{n+1} - p_n|}{|p_n - p|} = 1.$$

Reminder: A sequence $\{p_n\}$ is said to be superlinearly convergent to p if

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|} = 0.$$