Math 128A: Worksheet #7

Name: ______________________________ Date: March 10, 2021

Spring 2021

Problem 1 (4.2, #1a)**.** Apply Richardson's Extrapolation on the centered-difference formula:

$$f'(x_0) = \frac{1}{2h}[f(x_0+h) - f(x_0-h)] - \frac{h^2}{6}f'''(x_0) - \frac{h^4}{120}f^{(5)}(x_0) - \ldots$$

to determine $N_3(h)$, an approximation to $f'(x_0)$, for the following function and stepsize:

$$f(x) = \ln(x), \quad x_0 = 1.0, \quad h = 0.4.$$

Problem 2. Consider the following numerical integration rule:

$$\int_a^b f(x)\,dx \approx (b-a)\left(\frac{1}{4}f(a) + \frac{3}{4}f\left(a + \frac{2}{3}(b-a)\right)\right)$$

What is the degree of accuracy of this integration rule?
Hint: In order to make the computations simpler, you can assume without loss of generality that $a = 0$ *and* $b = 1$.

Problem 3. Consider a function $f : [0, 1] \to \mathbb{R}$. We want to approximate the integral $I = \int_0^1 f(x)\, dx$ using composite numerical integration based on the above integration rule. Let $I(h)$ denote the approximation of I we obtain by dividing the interval $[0, 1]$ into subintervals of length h. What is the order of the error $|I - I(h)|$ as $h \to 0$, i.e. what is the largest integer k such that

$$|I - I(h)| = \mathcal{O}(h^k) \text{ as } h \to 0$$

Hint: In each of the small subintervals of length h approximate f by a Taylor polynomial and use the degree of accuracy determined in Problem 1.