Math 128A: Worksheet #10

Name: ____________________ Date: April 14, 2021

Spring 2021

Problem 1. Derive the Adams-Moulton two-step method using divided differences for the interpolating polynomial.

Problem 2 (5.7, #1a)**.** Use the Adams Variable Step-Size Predictor-Corrector Algorithm with tolerance $TOL = 10^{-4}$, $hmax = 0.25$, and $hmin = 0.025$ to approximate the solutions to the given initial-value problem. Compare the results to the actual values.

$$y' = te^{3t} - 2y, \quad 0 \leq t \leq 1, \quad y(0) = 0; \quad \text{actual solution } y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

Problem 3. Consider the second order initial value problem

$$\begin{cases} y''(t) + \sin(y'(t)) + y(t)^2 = t^2 \\ y(0) = 1 \\ y'(0) = \pi/2 \end{cases}$$

1. Convert this second order equation into a first order system of equations.
2. Apply one step of Euler's method with step size h to this first order system.

Problem 4 (5.10, #4-ish)**.** Consider the following multistep method to solve the differential equation:

$$w_{i+1} = 5w_i - 4w_{i-1} - 3hf(t_{i-1}, w_{i-1}).$$

Analyze this method for consistency, stability, and convergence.

Problem 5 (5.10, #7)**.** Investigate stability for the difference method

$$w_{i+1} = -4w_i + 5w_{i-1} + 2h[f(t_i, w_i) + 2hf(t_{i-1}, w_{i-1})],$$

for $i = 1, 2, \ldots, N-1$, with starting values w_0, w_1.

Problem 6. Find the region of absolute stability (RAS) for the midpoint method:

$$w_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}f(t_i, w_i)\right).$$

Plot the RAS using Matlab.