

Math 128A: Worksheet #12

Name: _____ Date: April 28, 2021

Spring 2021

Problem 1. Show that the product of two $n \times n$ lower-triangular matrices is lower triangular.

Problem 2. Show that the inverse of a non-singular $n \times n$ lower-triangular matrix is lower triangular.

Problem 3. Use mathematical induction to show that when $n > 1$, the evaluation of the determinant of an $n \times n$ matrix using the definition requires

$$n! \sum_{k=1}^{n-1} \frac{1}{k!} \text{ multiplications/divisions} \quad \text{and} \quad n! - 1 \text{ additions/subtractions.}$$

- Problem 4.**
1. Show that solving $Ax = b$ by first factoring into $A = LU$ and then solving $Ly = b$ and $Ux = y$ requires the same number of operations as the Gaussian Elimination Algorithm 6.1
 2. Count the number of operations required to solve m linear systems $Ax^{(k)} = b^{(k)}$ for $k = 1, \dots, m$ by first factoring A and then using the method of part (c) m times. Compare this to doing Gaussian Elimination m times.

Problem 5. MATLAB demo of LU factorizations and how pivoting is ingrained in the `lu(A)`.

Problem 6 (6.6 #17). Find all α so that $A = \begin{bmatrix} 2 & \alpha & -1 \\ \alpha & 2 & 1 \\ -1 & 1 & 4 \end{bmatrix}$ is positive definite.