

Math 54: Worksheet #1

Name: _____ Date: August 31, 2021

Fall 2021

Problem 1 (True/False). One vector in \mathbb{R}^2 can span \mathbb{R}^2 .

Problem 2 (True/False). Any two vectors in \mathbb{R}^2 can span \mathbb{R}^2 .

Problem 3 (True/False). The columns of an $m \times n$ matrix A span \mathbb{R}^m iff there is a pivot in each row of $\text{REF}(A)$.

Problem 4 (True/False). Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

The system $Ax = b$ has a unique solution for any b .

Problem 5 (True/False). A system $Ax = b$ has a solution if and only if b is in the span of the columns of A .

Problem 6 (True/False). Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x .

Problem 7 (1.3 #6). Write a system of equations that is equivalent to the following vector equation:

$$x_1 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Problem 8 (1.3 #26). Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$ and let $b = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$. Let W be the set of all linear combinations of the columns of A .

1. Is b in W ?
2. Show that the third column of A is in W ?

Problem 9 (1.4 #20). Let

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}.$$

Can every vector in \mathbb{R}^4 be written as a linear combination of the columns of the matrix B above? Do the columns of B span \mathbb{R}^4 ?