Math 54: Worksheet #2

Name: ______________________________ Date: September 2, 2021

Fall 2021

Problem 1 (True/False)**.** If $\mathbf{p}$ is a solution to $A\mathbf{x} = \mathbf{b}$ and $\mathbf{u}$ is a solution to $A\mathbf{x} = \mathbf{0}$, then $\mathbf{v} = \mathbf{p} + \mathbf{u}$ is a solution to $A\mathbf{x} = \mathbf{b}$.

Problem 2 (True/False)**.** If the system $A\mathbf{x} = \mathbf{0}$ has 3 free variables, then there exists vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ such that the solution set is

$$\mathbf{x} = r\mathbf{v}_1 + s\mathbf{v}_2 + t\mathbf{v}_3, \quad (r, s, t \text{ in } \mathbb{R})$$

Problem 3 (True/False)**.** The columns of a matrix A are linearly *dependent* if and only if the homogeneous system $A\mathbf{x} = \mathbf{0}$ has only one solution. (What do we need in terms of pivots for the second half to be true?)

Problem 4 (True/False)**.** Let $\mathbf{v}, \mathbf{u}, \mathbf{w} \in \mathbb{R}^m$. Suppose that the set $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent, the set $\{\mathbf{v}, \mathbf{w}\}$ is linearly independent, and the set $\{\mathbf{u}, \mathbf{w}\}$ is linearly independent. Then, the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent.

Problem 5 (1.5 #10)**.** Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row equivalent to the following matrix:

$$\begin{bmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -8 \end{bmatrix}$$

Problem 6 (1.5 #18)**.** Describe and compare the solution sets of $x_1 - 3x_2 + 5x_3 = 0$ and $x_1 - 3x_2 + 5x_3 = 4$.

Problem 7 (1.7 #12)**.** Find the values of h for which the vectors are linearly *dependent.*

$$\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 7 \\ -3 \end{bmatrix}, \begin{bmatrix} 8 \\ h \\ 4 \end{bmatrix}$$

Problem 8 (1.7 #24)**.** Describe the possible echelon forms of a matrix A, if A is a 2×2 matrix with linaerly depedent columns.