Math 54: Worksheet #4

Name: _____ Date: September 9, 2021

Fall 2021

Problem 1 (True/False)**.** A linear map $T : \mathbb{R}^3 \to \mathbb{R}^2$ can be a surjection.

Problem 2 (True/False)**.** A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effect on all the coordinate vectors $\underline{e}_1, \ldots, \underline{e}_n$ in $\mathbb{R}^n$.

Problem 3 (True/False)**.** If A is a 4×3 matrix, the transformation $\underline{x} \to A\underline{x}$ cannot be one-to-one.

Problem 4 (True/False)**.** A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each vector in $\mathbb{R}^n$ maps to a unique vector in $\mathbb{R}^m$.

Problem 5 (1.8 #10)**.** Find the standard matrix of the following linear transformation: $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the vertical x_2-axis and then rotates points $\pi/2$ radians.

Problem 6 (1.8 #16-ish)**.** Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^3$ is given by

$$T(\underline{x}) = T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ 3x_1 + 5x_2 \\ x_1 \end{bmatrix}.$$

Find a 3×2 matrix such that $T(\underline{x}) = A\underline{x}$

Problem 7. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ such that $T(\underline{x}) = A\underline{x}$, where

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 2 & 0 \\ 0 & 6 & 8 \\ 6 & 9 & 1 \end{bmatrix}.$$

Is the transformation one-to-one? Is it onto?