

# Math 54: Worksheet #8

Name: \_\_\_\_\_ Date: September 28, 2021

Fall 2021

**Problem 1** (True/False). For any finite subset  $S$  of a vector space  $V$ ,  $\text{span}(S)$  is a subspace of  $V$ .

**Problem 2** (True/False). The integers  $\{\dots, -2, -1, 0, 1, 2, \dots\} \subset \mathbb{R}$  are a subspace of  $\mathbb{R}$ .

**Problem 3** (True/False).  $\text{Col } A$  is the set of all solutions of  $A\underline{x} = \underline{b}$ .

**Problem 4** (True/False). The range of a linear transformation is a vector space.

**Problem 5** (4.1 #10). Let  $H$  be the set of all vectors of the form  $\begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix}$ . Show that  $H$  is a subspace of  $\mathbb{R}^3$ .

**Problem 6** (4.1 #33). Given subspaces  $H$  and  $K$  of a vector space  $V$ , the sum of  $H$  and  $K$ , written as  $H + K$ , is the set of all vectors in  $V$  that can be written as the sum of two vectors, one in  $H$  and the other in  $K$ ; that is,

$$H + K = \{\underline{w} : \underline{w} = \underline{u} + \underline{v} \text{ for some } \underline{u} \text{ in } H \text{ and some } \underline{v} \text{ in } K\}$$

- (a) Show that  $H + K$  is a subspace of  $V$ .
- (b) Show that  $H$  is a subspace of  $H + K$  and  $K$  is a subspace of  $H + K$ .

**Problem 7** (4.2 #10). Determine if the given set,  $W$ , is a vector space, or find a specific example to the contrary:

$$\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{array}{l} a + 3b = c \\ b + c + a = d \end{array} \right\}$$

**Problem 8** (4.2 #24). Let  $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$  and  $\underline{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ . Determine if  $\underline{w}$  is in  $\text{Col } A$ . Is  $\underline{w}$  in  $\text{Nul } A$ ?