Math 54: Worksheet #9

Name: ______________________ Date: September 30, 2021

Fall 2021

Problem 1 (True/False)**.** A linearly independent set in a subspace H is a basis for H.

Problem 2 (True/False)**.** A basis is a linearly independent set that is as large as possible.

Problem 3 (True/False)**.** Suppose $\mathcal{B}$ is a basis of vector space V. The correspondence from $\mathbb{R}^n$ to V given by $[\underline{x}]_{\mathcal{B}} \mapsto \underline{x}$ is called the coordinate mapping.

Problem 4 (True/False)**.** A plane in $\mathbb{R}^3$ can be isomorphic to $\mathbb{R}^2$.

Problem 5 (4.3 #4)**.** Determine if the following set is a basis for $\mathbb{R}^3$. If it is not, determine if it linearly independent and/or if it spans $\mathbb{R}^3$:

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix}$$

Problem 6 (4.3 #14)**.** Consider the following matrix and one of its row-echelon forms:

$$A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}, \qquad REF(A) = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find bases for $\operatorname{Nul} A$ and $\operatorname{Col} A$.

Problem 7 (4.4 #8)**.** Find the coordinate vector $[\underline{x}]_{\mathcal{B}}$ of $\underline{x}$ relative to the given bases $\mathcal{B} = \{\underline{b}_1, \underline{b}_2, \underline{b}_3\}$:

$$\underline{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \underline{b}_2 = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}, \underline{b}_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \underline{x} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$$

Problem 8 (4.4 #29)**.** Use coordinate vectors to test the linear independence of the sets of polynomials:

$$(1-t)^2, t - 2t^2 + t^3, (1-t)^3$$