Math 54: Worksheet #10

Name: ______________________________ Date: October 5, 2021

Fall 2021

Problem 1 (True/False)**.** If a set $\{\underline{v}_1, \ldots, \underline{v}_p\}$ spans a finite-dimensional vector space V and if T is a set of more than p vectors in V, then T is linearly dependent.

Problem 2 (True/False)**.** For an $m \times n$ matrix A, the number of pivot columns equals the dimension of its null space, and the number of remaining columns (corresponding to free variables) equals the dimension of the column space.

Problem 3 (True/False)**.** The row space of A is the same as the column space of A^T.

Problem 4 (True/False)**.** For an $m \times n$ matrix A, the dimension of $\operatorname{Row} A$ is the same as the dimension of $\operatorname{Nul} A$.

Problem 5 (4.5 #5)**.** Consider the following subspace of $\mathbb{R}^4$:

$$\left\{ \begin{bmatrix} a-4b-2c \\ 2a+5b-4c \\ -a+2c \\ -3a+7b+6c \end{bmatrix} : a, b, c \text{ in } \mathbb{R} \right\}$$

(a) Find a basis of this subspace.

(b) State the dimension.

Problem 6 (4.5 #14)**.** Determine the dimensions of Nul A and Col A for the following matrix

$$A = \begin{bmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 0 & 0 & 1 & -3 & 7 & 0 \\ 0 & 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Problem 7 (4.6 #2)**.** Consider the matrices

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -3 & 0 & 5 & -7 \\ 0 & 0 & 2 & -3 & 8 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Assume that the matrix A is row equivalent to B. Without calculations, list $\operatorname{Rank} A$ and $\dim \operatorname{Nul} A$. Then, find bases for $\operatorname{Col} A$, $\operatorname{Row} A$, and $\operatorname{Nul} A$.