Math 54: Worksheet #11

Name: ______________________________ Date: October 7, 2021

Fall 2021

Problem 1 (True/False)**.** Suppose V and W are finite-dimensional vector spaces.If $T : V \to W$ is injective, then $\dim V \leq \dim W$.

Problem 2 (True/False)**.** Consider a finite-dimensional vector space V with two bases $\mathcal{C}$ and $\mathcal{B}$. Then, the columns of $P_{\mathcal{C}\leftarrow\mathcal{B}}$ are linearly independent.

Problem 3 (True/False)**.** Consider a finite-dimensional vector space V with two bases $\mathcal{C}$ and $\mathcal{B}$. Then,

$$P_{\mathcal{B}\leftarrow\mathcal{C}} = P_{\mathcal{C}\leftarrow\mathcal{B}}^{-1}.$$

Problem 4 (4.6 #6)**.** If a 6×3 matrix A has rank 3, find $\dim \operatorname{Nul} A$, $\dim \operatorname{Row} A$, and $\operatorname{Rank} A^T$.

Problem 5 (4.6 #25)**.** A scientist solves a nonhomogeneous system of ten linear equations in twelve unknowns and finds that three of the unknowns are free variables. Can the scientist be certain that, if the right sides of the equations are changed, the new nonhomogeneous system will have a solution? Discuss.

Problem 6 (4.7 #6)**.** Let $\mathcal{D} = \{\underline{d}_1, \underline{d}_2, \underline{d}_3\}$ and $\mathcal{F} = \{\underline{f}_1, \underline{f}_2, \underline{f}_3\}$ be bases for a vector space V, and suppose $\underline{f}_1 = 2\underline{d}_1 - \underline{d}_2 + \underline{d}_3$, $\underline{f}_2 = 3\underline{d}_2 + \underline{d}_3$ and $\underline{f}_3 = -3\underline{d}_1 + 2\underline{d}_3$.

(a) Find the change of coordinates matrix from $\mathcal{F}$ to $\mathcal{D}$.

(b) Find $[\underline{x}]_{\mathcal{D}}$ for $\underline{x} = \underline{f}_1 - 2\underline{f}_2 + 2\underline{f}_3$.

Problem 7 (4.7 #14)**.** In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis $\mathcal{B} = \{1 - 3t^2, 2 + t - 5t^2, 1 + 2t\}$ to the standard basis. Then write t^2 as a linear combination of the polynomials in $\mathcal{B}$.