Math 54: Worksheet #14

Name: ____________________________________ Date: October 19, 2021

Fall 2021

Problem 1 (True/False)**.** If A is similar to B, then A^2 is similar to B^2.

Problem 2 (True/False)**.** If $B = P^{-1}AP$ and $\underline{x}$ is an eigenvector of A corresponding to an eigenvalue λ, then $P\underline{x}$ is an eigenvector of B corresponding to λ.

Problem 3 (True/False)**.** If $A = PCP^{-1}$, then C is the $\mathcal{B}$-matrix for the transformation $\underline{x} \mapsto A\underline{x}$ when $\mathcal{B}$ is the basis formed by the columns of P.

Problem 4 (5.4 #4)**.** Let $\mathcal{B} = \{\underline{b}_1, \underline{b}_2, \underline{b}_3\}$ be a basis for a vector space V and $T : V \to \mathbb{R}^2$ be a linear transformation with the property that

$$T(x_1\underline{b}_1 + x_2\underline{b}_2 + x_3\underline{b}_3) = \begin{bmatrix} 2x_1 - 4x_2 + 5x_3 \\ -x_2 + 3x_3 \end{bmatrix}.$$

Find the matrix for T relative to $\mathcal{B}$ and the standard basis for $\mathbb{R}^2$.

Problem 5 (5.4 #6)**.** Let $T : \mathbb{P}_2 \to \mathbb{P}_4$ be the transformation that maps a polynomial $p(t)$ into the polynomial $p(t) + t^2 p(t)$.

a. Find the image of $p(t) = 2 - t + t^2$.

b. Show that T is a linear transformation.

c. Find the matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3, t^4\}$.

Problem 6 (5.4 #16). Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\underline{x}) = A\underline{x}$, where

$$A = \begin{bmatrix} 2 & -6 \\ -1 & 3 \end{bmatrix}$$

Find a basis $\mathcal{B}$ for $\mathbb{R}^2$ with the property that $[T]_{\mathcal{B}}$ is diagonal.