Math 54: Worksheet #15

Name: ______________________________ Date: October 21, 2021

Fall 2021

Problem 1 (True/False)**.** Over $\mathbb{C}$, any $n \times n$ matrix (with real or complex entries) has an eigenvalue.

Problem 2 (True/False)**.** Over $\mathbb{C}$, any $n \times n$ matrix (with real or complex entries) is diagonalizable.

Problem 3 (True/False)**.** For a real $n \times n$ matrix, if λ is an eigenvalue, then $\bar{\lambda}$ is an eigenvalue.

Problem 4 (5.5 #4)**.** Consider the following matrix (acting on $\mathbb{C}^2$):

$$\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}.$$

Find the eigenvalues, and a basis for each eigenspace in $\mathbb{C}^2$. Can you diagonalize A?

Problem 5 (5.5 #10)**.** Consider the following matrix (acting on $\mathbb{C}^2$):

$$A = \begin{bmatrix} -5 & -5 \\ 5 & -5 \end{bmatrix}.$$

The transformation $\underline{x} \mapsto A\underline{x}$ is the composition of a rotation and a scaling. Give the angle φ of the rotation, where $-\pi < \varphi \leq \pi$, and give the scale factor r.

Problem 6 (5.5 #16)**.** Consider the following matrix (acting on $\mathbb{C}^2$):

$$\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}.$$

Use the information in Exercise 4 to find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that the given matrix has the form $A = PCP^{-1}$.