Math 54: Worksheet #16

Name: ______________________________ Date: October 26, 2021

Fall 2021

Problem 1 (True/False)**.** If $W = \operatorname{Col} A$, then $W^\perp = \operatorname{Nul} A$.

Problem 2 (True/False)**.** Not every orthogonal set in $\mathbb{R}^n$ is linearly independent.

Problem 3 (True/False)**.** Suppose $\mathcal{U} = \{\underline{u}_1, \dots, \underline{u}_n\}$ is an orthonormal basis of $\mathbb{R}^n$. Then, for any $\underline{v} \in \mathbb{R}^n$,

$$[\underline{v}]_{\mathcal{U}} = \begin{bmatrix} \underline{v} \cdot \underline{u}_1 \\ \vdots \\ \underline{v} \cdot \underline{u}_n \end{bmatrix}.$$

Problem 4 (True/False)**.** Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear and preserves lengths (i.e., for each $\underline{x} \in \mathbb{R}^n$, $\|T(\underline{x})\| = \|\underline{x}\|$), then T must be injective.

Problem 5 (6.1 #6)**.** Consider the vectors

$$\underline{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}.$$

Compute $\left(\dfrac{\underline{x} \cdot \underline{w}}{\underline{x} \cdot \underline{x}}\right) \underline{x}$.

Problem 6 (6.1 #29)**.** Let $W = \text{span}\{\underline{v}_1, \ldots, \underline{v}_p\}$. Show that if $\underline{x}$ is orthogonal to each $\underline{v}_j$, for $1 \leq j \leq p$, then $\underline{x}$ is in $W^{\perp}$.

Problem 7 (6.2 #10)**.** Consider the following vectors:

$$\underline{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \quad \underline{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \quad \underline{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}.$$

Show that $\{\underline{u}_1, \underline{u}_2, \underline{u}_3\}$ is an orthogonal basis of $\mathbb{R}^3$. Then, express $\underline{x}$ as a linear combinations of the $\underline{u}$'s.

Problem 8 (6.2 #20)**.** Consider the following vectors:

$$\begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix}.$$

Determine if the set of vectors are orthonormal. If the set is only orthogonal, normalize the vectors to produce an orthonormal set.