Math 54: Worksheet #17

Name: ______________________________ Date: October 28, 2021

Fall 2021

Problem 1 (True/False)**.** If $\underline{y} = \underline{z}_1 + \underline{z}_2$ where $\underline{z}_1$ is in W and $\underline{z}_2$ is in $W^\perp$, then $\underline{z}_1$ must be the orthogonal projection of $\underline{y}$ onto W.

Problem 2 (True/False)**.** If an $n \times p$ matrix U has orthonormal columns, then $UU^T\underline{x} = \underline{x}$ for all $\underline{x}$ in $\mathbb{R}^n$.

Problem 3 (True/False)**.** If $\mathcal{B}$ is an eigenbasis of $\mathbb{R}^n$ for an $n \times n$ matrix A, then Gram-Schmidt of $\mathcal{B}$ gives an orthonormal eigenbasis of A.

Problem 4 (True/False)**.** If $W = \text{span}\{\underline{x}_1, \underline{x}_2, \underline{x}_3\}$ with $\{\underline{x}_1, \underline{x}_2, \underline{x}_3\}$ linearly independent, and if $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ is an orthogonal set in W, then $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ is a basis for W.

Problem 5 (6.3 #10)**.** Consider the following vectors:

$$\underline{y} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}, \quad \underline{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad \underline{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \underline{u}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

Let W be the subspace spanned by the $\underline{u}$'s, and write $\underline{y}$ as a sum of a vector in W and a vector orthogonal to W.

Problem 6 (6.3 #12)**.** Consider the following vectors:

$$\underline{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}, \quad \underline{v}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}, \quad \underline{v}_2 = \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}.$$

Find the closest point to $\underline{y}$ in the subspace W spanned by $\underline{v}_1$ and $\underline{v}_2$. Also, find the distance $\underline{y}$ to W.

Problem 7 (6.4 #10)**.** Consider the matrix

$$A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}.$$

Find an orthonormal basis of the $\operatorname{Col} A$. Explain how you would use this to factor $A = QR$.