Math 54: Worksheet #18

Name: ______________________________ Date: November 4, 2021

Fall 2021

Problem 1 (True/False)**.** If $\underline{b}$ is in the column space of A, then every solution of $A\underline{x} = \underline{b}$ is a least-square solution.

Problem 2 (True/False)**.** The least-squares solution of $A\underline{x} = \underline{b}$ is the point in the column space of A closest to $\underline{b}$.

Problem 3 (True/False)**.** The function $\langle f, g\rangle = f(0)^2 + f(1)^2 + g(0)^2 + g(1)^2$ is an inner product on the vector space $V = \mathbb{P}_2$, the space of polynomials of degree at most 2.

Problem 4 (True/False)**.** The function $\langle f, g\rangle = f(-2)g(-2) + f(0)g(0) + f(2)g(2)$ is an inner product on the vector space $V = \mathbb{P}_2$, the space of polynomials of degree at most 2.

Problem 5 (6.5 #10)**.** Consider the following matrix A and vector $\underline{b}$:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}.$$

(a) Find the orthogonal projection of $\underline{b}$ onto $\operatorname{Col} A$.

(b) Find a least squares solution of $A\underline{x} = \underline{b}$.

Problem 6 (6.6 #4)**.** Find the line of best fit, $y = \beta_0 + \beta_1 x$, which minimizes the square of the difference in y-values for the following data points:

$$(2,3),\ (3,2),\ (5,1)\ (6,0).$$

Problem 7 (6.7 #9)**.** Let $\mathbb{P}_3$ have the inner product given by evaluation at -3, -1, 1, and 3: $\langle f, g\rangle = f(-3)g(-3) + f(-1)g(-1) + f(1)g(1) + f(3)g(3)$. Let $p_0(t) = 1$, $p_1(t) = t$, and $p_2(t) = t^2$.

(a) Compute the orthogonal projection of p_2 onto the subspace spanned by p_0 and p_1.

(b) Find a polynomial q that is orthogonal to p_0 and p_1 such that $\{p_0, p_1, q\}$ is orthogonal basis for $\operatorname{span}\{p_0, p_1, p_2\} = \mathbb{P}_2$. Scale the polynomial q so that its vector of values at $(-3, -1, 1, 3)$ is $(1, -1, -1, 1)$.