Math 54: Worksheet #19

Name: ______________________________ Date: November 9, 2021

Fall 2021

Problem 1 (True/False)**.** All real symmetric matrices are diagonalizable over $\mathbb{R}$.

Problem 2 (True/False)**.** Eigenspaces of a real symmetric matrix are mutually orthogonal.

Problem 3 (True/False)**.** A quadratic form Q on $\mathbb{R}^n$ corresponds to a unique real symmetric matrix A by $Q(\underline{x}) = \underline{x}^T A \underline{x}$.

Problem 4 (True/False)**.** The eigenvalues of $A^T A$ and AA^T are real and non-negative.

Problem 5 (True/False)**.** If A is square ($n \times n$) and invertible with SVD $A = U\Sigma V^T$, then $A^{-1} = V\Sigma U^T$.

Problem 6 (True/False)**.** If A is $n \times n$ and symmetric, then the singular values of A coincide with the eigenvalues of A.

Problem 7 (7.1 #18)**.** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -6 & 4 \\ -6 & 2 & -2 \\ 4 & -2 & -3 \end{bmatrix}.$$

The eigenvalues are $-3, -6, 9$. Orthogonally diagonalize the matrix, giving an orthogonal matrix P and a diagonal matrix D.

Problem 8 (7.2 #8)**.** Let A be the matrix of the quadratic form

$$9x_1^2 + 7x_2^2 + 11x_3^2 - 8x_1x_2 + 8x_1x_3.$$

It can be shown that the eigenvalues of A are 3, 9, and 15. Find an orthogonal matrix P such that the change of variable $\underline{x} = P\underline{y}$ transforms $\underline{x}^T A\underline{x}$ into a quadratic form with no cross-product term. Give P and the new quadratic form.

Problem 9 (7.2 #20)**.** What is the largest value of the quadratic form $5x_1^2 - 3x_2^2$ if $\underline{x}^T\underline{x} = 1$?

Problem 10 (7.3 #10)**.** Find an SVD of the following matrix:

$$A = \begin{bmatrix} 7 & 1 \\ 5 & 5 \\ 0 & 0 \end{bmatrix}$$