Math 54: Worksheet #22

Name: _____   Date: November 23, 2021

Fall 2021

Problem 1 (True/False)**.** Every n-th order linear differential equation can be written as a first order system of linear differential equations (with n variables.)

Problem 2 (True/False)**.** Consider the following nonhomogeneous system of differential equations in normal form: $\underline{x}'(t) = A(t)\underline{x}(t) + \underline{f}(t)$. If $\underline{x}_p$ is a particular solution of the nonhomogeneous system and $\{\underline{x}_1, \ldots, \underline{x}_n\}$ form a fundamental solution set of the homogeneous system, then the general form of the solution to the nonhomogeneous system is

$$\underline{x}_p + c_1\underline{x}_1 + \cdots + c_n\underline{x}_n.$$

Problem 3 (9.1 #11). Express the following system of higher-order differential equations as a matrix system in normal form:

$$x'' + 3x + 2y = 0,$$
$$y'' - 2x = 0.$$

Problem 4 (9.4 #14). Determine whether the given vector functions are linearly dependent or linearly independent on the interval $(-\infty, \infty)$:

$$\begin{bmatrix} te^{-t} \\ e^{-t} \end{bmatrix}, \quad \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix}.$$

Problem 5 (9.4 #24). The following vector functions are solutions to a system $\underline{x}'(t) = A\underline{x}(t)$:

$$\begin{bmatrix} e^t \\ e^t \\ e^t \end{bmatrix}, \quad \begin{bmatrix} \sin t \\ \cos t \\ -\sin t \end{bmatrix}, \quad \begin{bmatrix} -\cos t \\ \sin t \\ \cos t \end{bmatrix}.$$

Determine whether they form a fundamental solution set. If they do, find a fundamental matrix for the system and give a general solution.