Math 54: Worksheet #23

Name: _____ Date: November 30, 2021

Fall 2021

Problem 1 (True/False)**.** For an $n \times n$ matrix A, the solution space of $\underline{x}' = A\underline{x}$ is n dimensional.

Problem 2 (True/False)**.** If $\underline{v}$ is an eigenvector of an $n \times n$ matrix A with eigenvalue λ, then $\underline{x} = e^{\lambda t}\underline{v}$ is a solution of $\underline{x}' = A\underline{x}$.

Problem 3 (9.5 #14)**.** Find a general solution of the system $\underline{x}' = A\underline{x}$, where

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix}.$$

Hint: the eigenvalues of A are -2, -1, and 3.

Problem 4 (9.5 #32)**.** Solve the following initial value problem:

$$\underline{x}' = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix} \underline{x}, \quad \underline{x}(0) = \begin{bmatrix} -10 \\ -6 \end{bmatrix}.$$

Problem 5 (9.6 #7)**.** Find a fundamental matrix for the system $\underline{x}' = A\underline{x}$, where

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Problem 6 (9.6 #13a-ish)**.** Solve the following initial value problem:

$$\underline{x}' = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \underline{x}, \quad \underline{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$