Math 54: Worksheet #24

Name: ______________________________ Date: December 2, 2021

Fall 2021

Problem 1 (True/False)**.** The function $f(x) = \sin^2 x$ is an odd function.

Problem 2 (True/False)**.** The function $f(x) = x^2 + \cos x$ is an even function.

Problem 3. Check that:

$$\int_{-L}^{L} \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right)\, dx = \begin{cases} 0 & \text{if } m \neq n, \\ L & \text{if } m = n \neq 0, \\ 2L & \text{if } m = n = 0. \end{cases}$$

Problem 4 (10.3 #12-ish)**.** Compute the Fourier series of

$$f(x) = \begin{cases} 0, & -\pi < x < 0, \\ x^2, & 0 < x < \pi. \end{cases}$$

Determine the function that the Fourier series converges to.

Problem 5 (10.4 #2)**.** Let $f(x) = \sin 2x$ for $0 < x < \pi$. Determine

1. the π-periodic extension.
2. the odd 2π-periodic extension.
3. the even 2π-periodic extension.

Problem 6 (10.4 #6-ish)**.** Compute the Fourier sine series and the Fourier cosine series for the function

$$f(x) = \cos x, \quad 0 < x < \pi.$$

Problem 7 (10.4 #18)**.** Let $f(x) = x(\pi - x)$. Find the solution to the following heat flow problem

$$\begin{aligned} \frac{\partial u}{\partial t} &= 5\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0, \\ u(0,t) &= u(\pi,t) = 0, \quad t > 0, \\ u(x,0) &= f(x), \quad 0 < x < \pi. \end{aligned}$$