Instructions:

- Your submission will consist of six files (and nothing else):
  - `bisection.m`
  - `newton.m`
  - `findbracket.m`
  - `newtonbisection.m`
  - `results1.txt`
  - `results2.txt`
- Very Important: Create a single compressed (.zip) folder with these files. Name it LastNameFirstNameProject, e.g. HeinzMichaelProject.zip

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In this assignment, we will address two issues with the Bisection method and Newton's method:

- Finding an interval $[a, b]$ for the Bisection method, with $f(a)$ and $f(b)$ having different signs.
- Combining the excellent convergence properties of Newton's method with the guaranteed root-finding (robustness) of the Bisection method.

1a (Bisection): Implement the Bisection Method for Root Finding.

```
function p = bisection(f, a, b, tol)
```

- `p`: approximation to the root
- `f`: function handle
- `a`: left endpoint of initial interval
- `b`: right endpoint of initial interval
- `tol`: absolute error tolerance for root

In particular, your code should return an error if `f(a)` and `f(b)` have the same sign, in which case the intermediate value theorem does not guarantee bisection can successfully find a root.

1b (Newton): Implement Newton's Method for Root Finding

```
function p = newton(f, df, p0, tol)
```

- `p`: approximation to the root

- `f`: function handle
- `df`: function handle of derivative
- `p0`: initial guess
- `tol`: absolute error tolerance for root

2. Implement a MATLAB function `findbracket` with signature

```
function [a, b] = findbracket(f, x0)
```

which finds an interval $[a, b]$ around x_0 such that $f(a)f(b) < 0$ (i.e. $f(a)$ and $f(b)$ have opposite signs) according to the following method:

1. Set $a = b = x_0$ and $dx = 0.001$
2. Set $a = a - dx$. If $f(a)f(b) < 0$, terminate.
3. Set $b = b + dx$. If $f(a)f(b) < 0$, terminate.
4. Multiply dx by 2 and repeat from step 2.

3. Implement a MATLAB function `newtonbisection` with signature

```
function p = newtonbisection(f, df, a, b, tol)
```

combining Newton's method and the Bisection method according to the following strategy:

1. Start with $p = a$
2. Attempt a Newton step $p = p - \dfrac{f(p)}{f'(p)}$
3. If p is outside of $[a, b]$, set $p = \dfrac{a+b}{2}$
4. If $f(p)f(b) < 0$, set $a = p$, otherwise set $b = p$.
5. Terminate if $|f(p)| <$ `tol`
6. Repeat from step 2.

This function will be like a combination of `newton` and `bisection`.

4. Run your function `newtonbisection` using $f(x) = \sin(x) - e^{-x}$ on the interval $[1.9, 30]$:

```
f = @(x) sin(x) - exp(-x);
df = @(x) cos(x) + exp(-x);
```

```
x = newtonbisection(f, df, 1.9, 30, 1e-8)
```

Present the result in a table showing for each iteration the method used (Newton or Bisection), a, b, p, and $f(p)$ (*Hint:* it is easiest to print this information each iteration inside the function `newtonbisection`). Save this as `results1.txt`. You don't need to do anything fancy or write a function to get the requested data, just temporarily print/display some of the results and format them in a text file.

5. Use your combined `findbracket` and `newtonbisection` to solve for the roots of $f(x) = \sin(x) - e^{-x}$ with $x_0 = -3, -2, -1, ..., 9, 10$:

```
f = @(x) sin(x) - exp(-x);
df = @(x) cos(x) + exp(-x);
for x0 = -3:10
   [a, b] = findbracket(f, x0);
   x = newtonbisection(f, df, a, b, 1e-8);
   [x0, a, b, x]
end
```

Present your results in a table showing $x_0, a, b,$ and x. Save this as `results2.txt`.